

$$D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu$$

Transport Theory for Non-Abelian Plasmas

$$D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu$$

Bridging Microscopic Dynamics and Effective Action

$$D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu$$

Based on lectures by Cristina Manuel

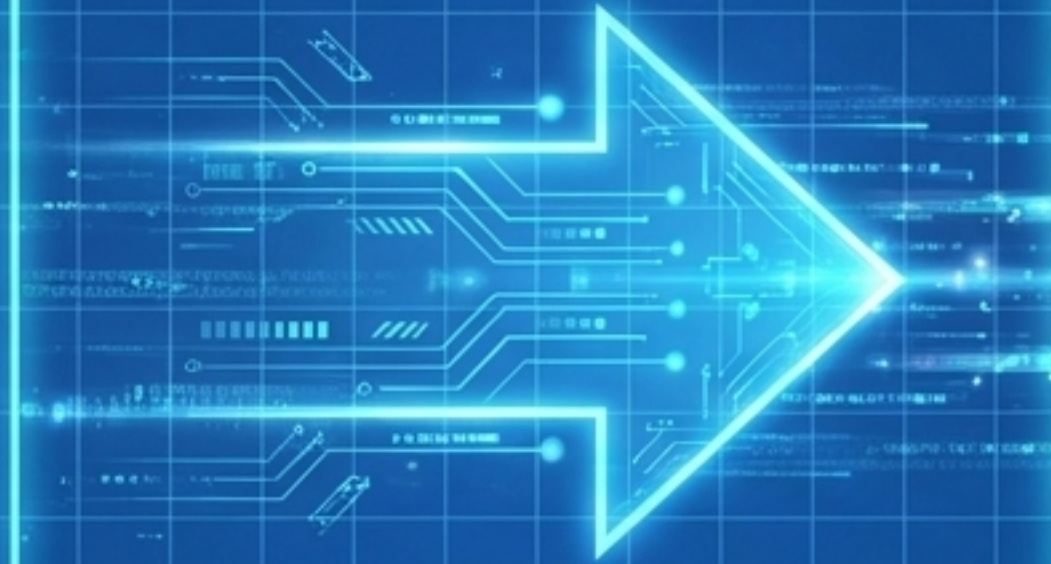
$$D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu \quad D_\mu F^{\mu\nu} = J^\nu$$

Microscopic Physics

$$f(x, p)$$

$$df/dt = C[f]$$

Quasiparticle
Picture Valid



Statistical
Averaging

Macroscopic Properties

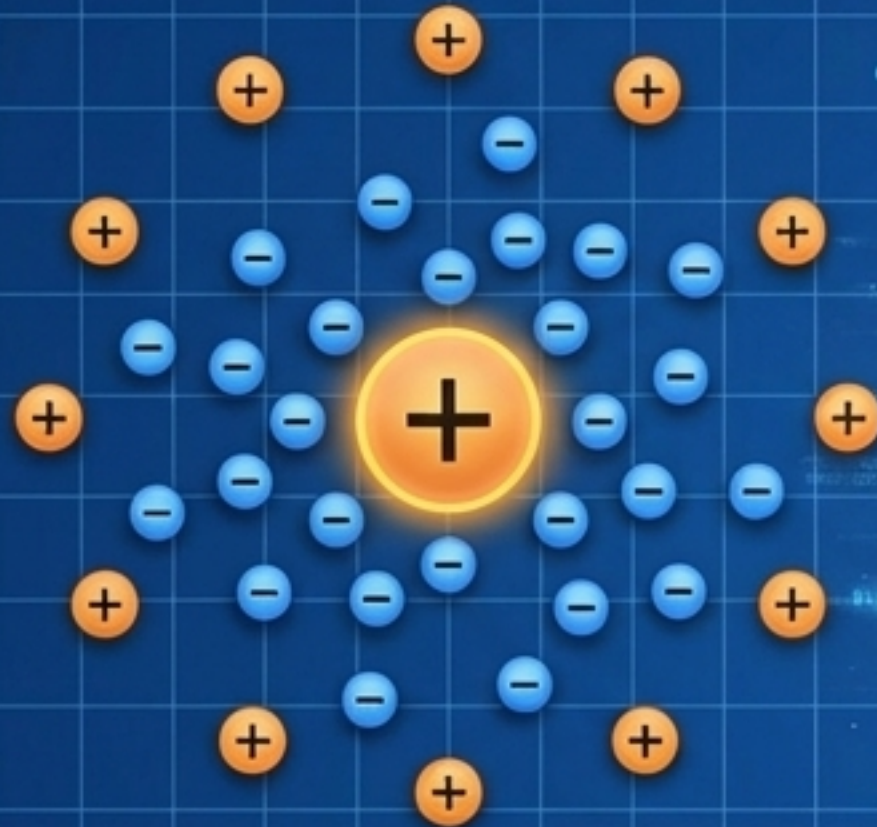
Stress-energy tensor:
 $\langle T^{\mu\nu} \rangle$

Color current:
 $\langle J^\mu \rangle$

Physical Regime: Unconfined phase of a non-Abelian theory where $g(T) \ll 1$ and $T \gg T_c$

Theoretical Blueprint

Debye Screening



$$\frac{1}{r} \rightarrow \frac{e^{-m_D r}}{r}$$

$$m_D^2 \propto e^2 T^2$$

Landau Damping



$$\frac{\partial(z, t)}{\partial t^2} + \frac{d^2 \psi(z, t)}{dt^2} = (z_f, T_f)$$

These classical kinetic phenomena (Silin, '60) must now be generalized to QCD.

Theoretical Blueprint

⚠ At $T \gg T_c$ and $g \ll 1$, naive perturbation theory breaks down. ⚠

Gauge Dependence

Gauge-dependent results emerge for gluonic damping rates.

Physical Observables

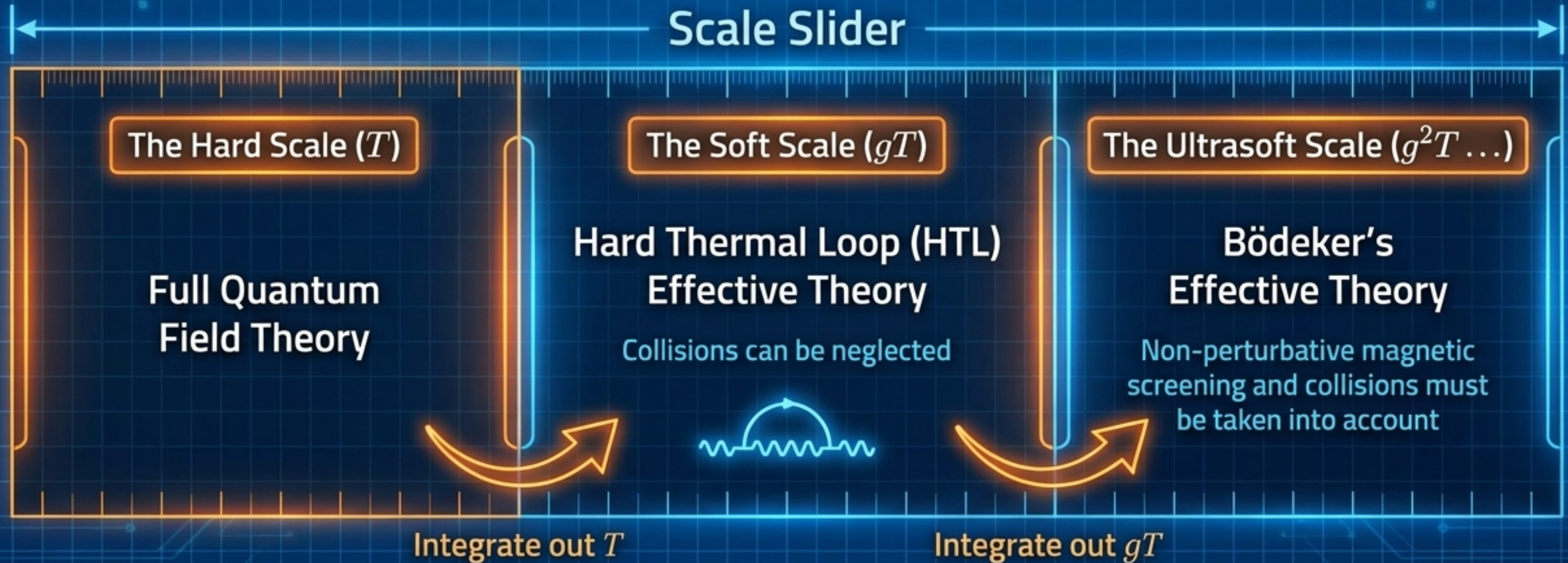
Non-Abelian observables are not always physically (e.g., color conductivity vs. electric conductivity).

IR Physics

Infrared (IR) physics is genuinely non-perturbative.

The Solution: Hard Thermal Loops (Braaten/Pisarski, '89)
One-loop thermal corrections are as important as tree amplitudes and must be resummed into effective propagators.

Theoretical Blueprint



The Microscopic Engine: Wong Equations (1970)

$$m \frac{dx^\mu}{d\tau} = p^\mu$$

$$m \frac{dp^\mu}{d\tau} = gQ^a F_a^{\mu\nu} p_\nu$$

Lorentz-like force
with color charge

$$m \frac{dQ^a}{d\tau} = -gf^{abc} p^\mu A_\mu^b Q^c$$

Color Dynamics: Color
charge Q^a rotates in
 $SU(N)$ space as it moves

Kinematics

Key Insight Box

Unlike QED, color charges are dynamical phase-space variables.

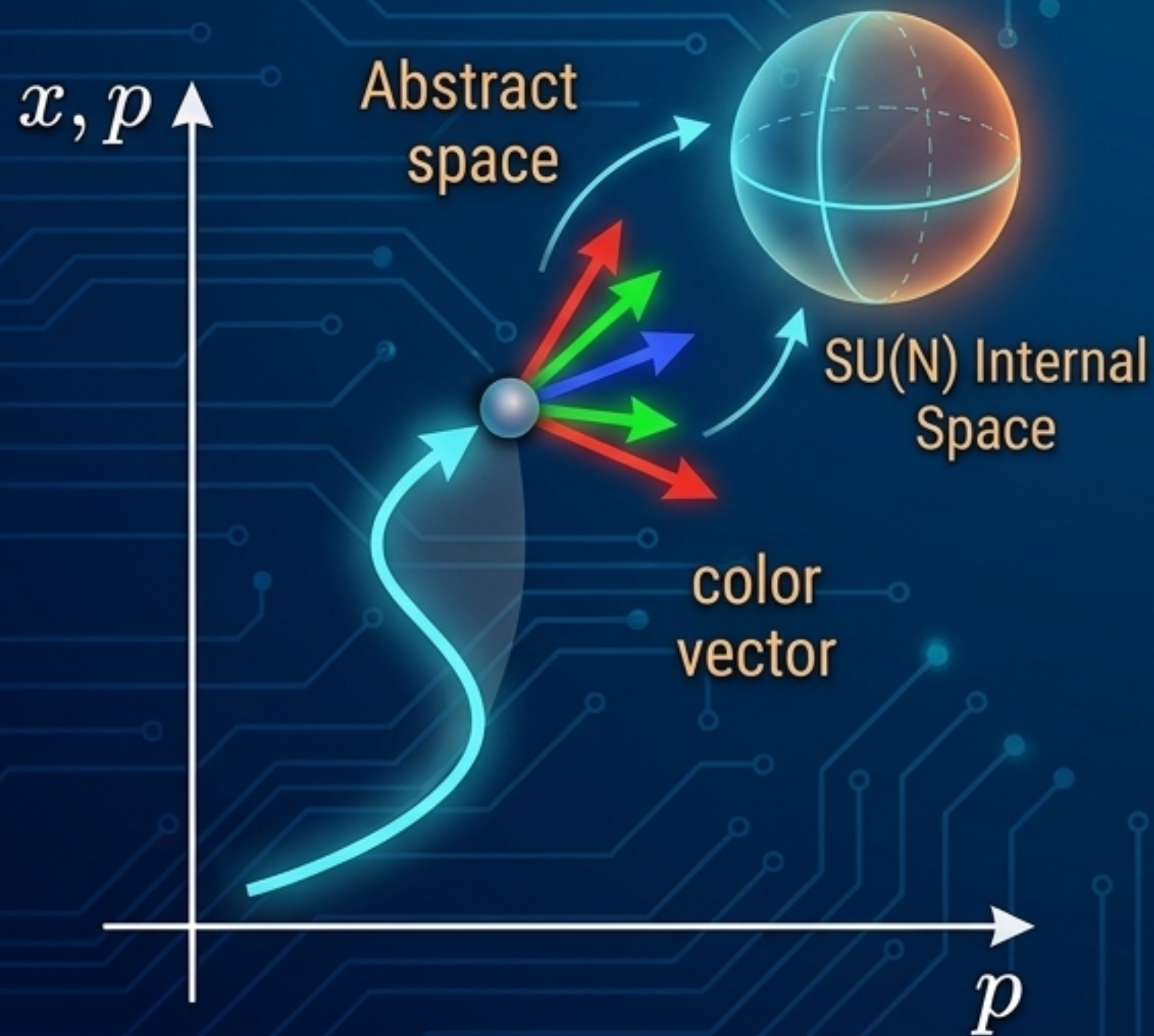
Refer to the microscopic approach phase-space details for context on these variables, indicating color charges evolve over time.

Theoretical Blueprint

Scale Slider



Location: Microscopic



$f(x^\mu, p^\mu, Q_a)$ = classical probability of finding a particle in this exact state

Constraint Checklist

- ☒ On-mass-shell evolution
 $p^2 - m^2 = 0$ and positive energy $\Theta(p_0)$
- ☒ Conservation of group invariants
e.g., for SU(3), $Q_a Q_a = c_2$ (quadratic Casimir)



Theoretical Blueprint

$$\underbrace{p^\mu D_\mu f}_{\text{Covariant Drift}} - \underbrace{g Q_a F_a^{\mu\nu}}_{\text{Force}} \frac{\partial f}{\partial p^\nu} = \underbrace{C[f]}_{\text{Collision Term}}$$

Covariant Drift

$$D_\mu = \partial_\mu - g f^{abc} Q_c A_\mu^b \frac{\partial}{\partial Q^a}$$

Force

Collision Term
(Zero in the Vlasov approximation)

Coupling to Yang-Mills fields

The system couples back to the Yang-Mills fields via the covariantly conserved current:

$$J_a^\mu(x) = g \int dP dQ p^\mu Q_a f$$

Theoretical Blueprint

Scale Slider

Location: Soft Scale (gT)

Emergence of Hard Thermal Loops (HTL)

Near-Equilibrium Expansion

$$f = f^{(0)} + g f^{(1)} + g^2 f^{(2)} \dots$$

where $f^{(0)}$ is the Fermi-Dirac/Bose-Einstein background.



The First Correction (g)

Linearized transport equation for $f^{(1)}$ in the presence of a weak external color field ($A \sim T$).



The HTL Effective Action

Integrating out the momentum yields the induced color current J_a^μ .

$$J_a^\mu(x) = -\frac{\delta \Gamma_{\text{HTL}}[A]}{\delta A_{\mu a}(x)}$$

Takeaway: Quantum HTLs can be understood as classical thermal fluctuations!

Dense Quark Matter: At low T and high μ , quarks condense ($\langle qq \rangle \neq 0$).

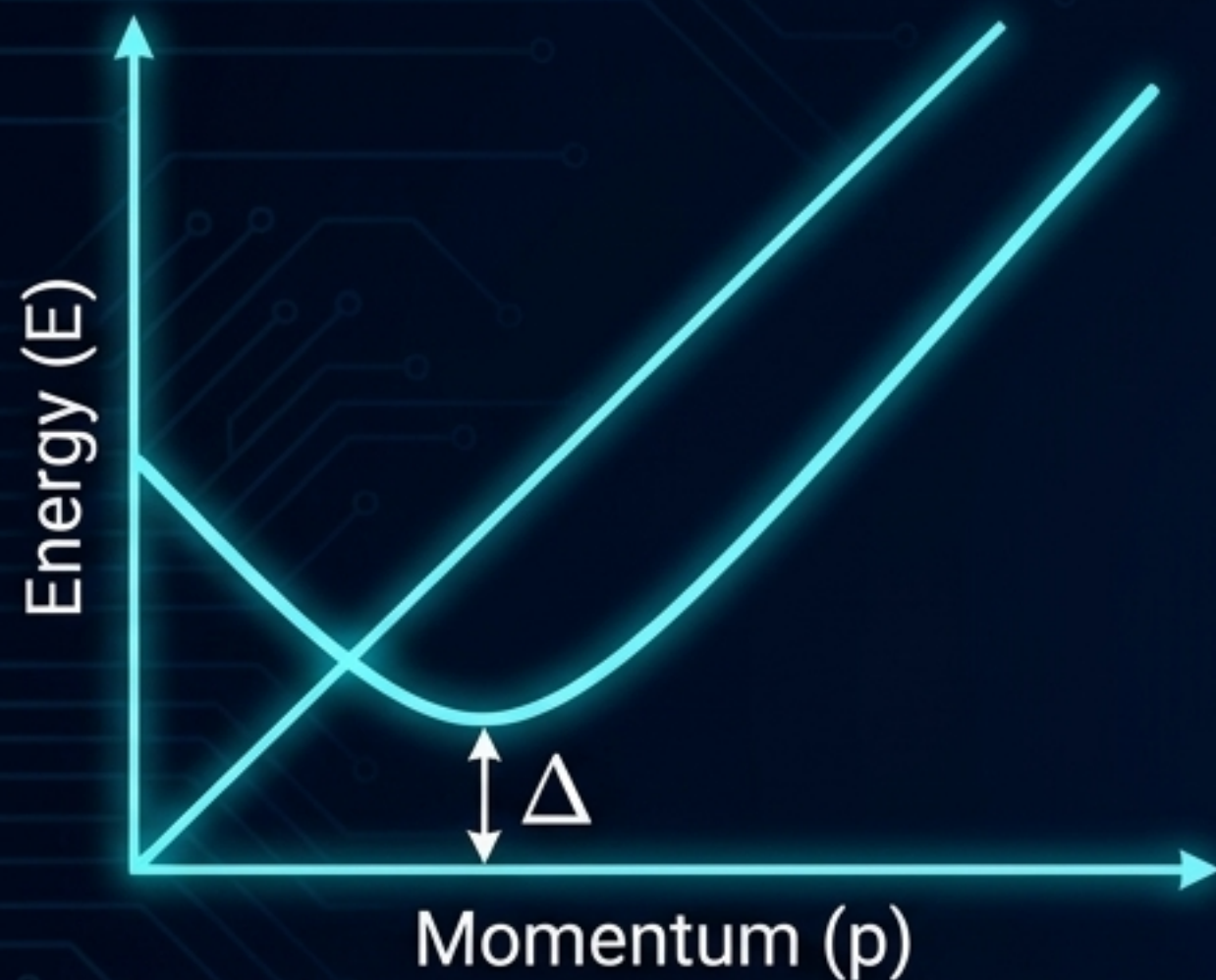
$N_f = 3$ (CFL)	$N_f = 2$ (2SC)
• Color-Flavor Locked phase. • All gluons acquire mass. • Long-distance physics is strictly color-neutral.	• Two-Flavor Color Superconductor. • Symmetry breaking: $SU(3)_c \rightarrow SU(2)_c$. • Result: 3 gluons remain massless at $T=0$. Gapless quark excitations exist.

Takeaway:

How do we write transport equations for the remaining color excitations in 2SC?

Transport in the 2SC Gapped State

Location: Dense Superconductors



- Gapped quasiparticles have a modified energy:

$$E_p = \sqrt{(p - \mu)^2 + \Delta^2}$$

- Key Consequence:

$$\text{Velocity } \vec{v}_p = \frac{\partial E_p}{\partial \vec{p}}$$

These particles do not travel at the speed of light ($|\vec{v}_p| \leq 1$).

$$\left[v_p^\mu D_\mu - gQ_a \left(\vec{E}^a + \vec{v}_p \times \vec{B}^a \right) \cdot \frac{\partial}{\partial \vec{p}} \right] f = C[f]$$

If $\Delta \rightarrow 0$, the equation perfectly reduces to standard QGP equations at soft scales.

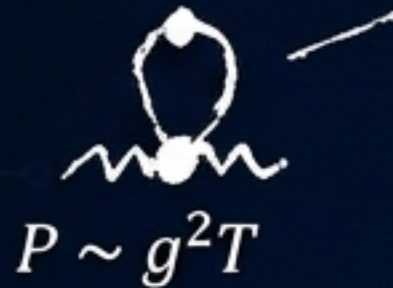
The Collisionless Approximation Fails

Scale Slider



Location: Ultrasoft Scale (g^2T)

Sym (Yang-Mills) + Γ_{HTL} (Soft internal lines, resummed)



soft internal lines (resummed)



ultra-soft external lines

Cutoff μ where $gT \ll \mu \ll g^2T$

Bödeker's Effective Theory (1998)

At the **ultrasoft scale** ($\gamma \sim g^2T \ln(1/g)$), the gluon damping rate forces us to take collisions into account. The resulting effective theory is **UV finite** and can be treated numerically.

Deconstructing Bödeker's Theory

Scale Slider



Location: Ultrasoft Scale ($g^2 T$)

$$(\mathbf{v} \cdot \mathbf{D}\mathbf{J})^a = \underbrace{m_D^2 \vec{v} \cdot \vec{E}^a}_{\text{The Mean Field Response}} + \underbrace{\delta C^a}_{\text{Linearized Collision Term}} + \underbrace{\xi^a}_{\text{Stochastic Noise}}$$

The Mean Field Response

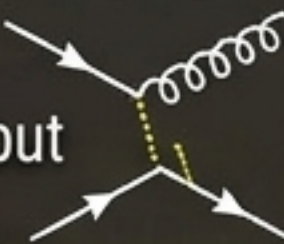
$$m_D^2 \vec{v} \cdot \vec{E}^a$$

Linearized Collision Term

$$\delta C^a$$

Dominated by forward scattering off Landau-damped magnetic gluons.

Local in space-time, but not in velocity space.



Stochastic Noise

$$\xi^a$$

Driven by statistical fluctuations in the plasma.

$$\begin{aligned} &\langle \xi_a(x, v) \xi_b(x', v') \rangle \\ &= 2T\gamma m_D^2 \delta_{ab} I(\vec{v}, \vec{v}') \delta^{(4)}(x - y) \end{aligned}$$



From Microstates to Macro Fluctuations

Scale Slider

Location: Ultrasoft Scale (g^2T)



1. Microstates

Integral over L identical particles.

Distribution function
 $\tilde{\mathcal{F}}(z_1, \dots, z_L; t) = 1$



2. Statistical Average

Taking the ensemble average over the microstates:

$$\langle G \rangle = \int dz_1 \dots dz_L G \tilde{\mathcal{F}}$$



3. Splitting

Separating mean from fluctuation:

$$N = \langle N \rangle + \delta N$$

where by definition
 $\langle \delta N \rangle = 0$



4. Correlators

Emergence of two-particle correlation $g_2(z, z')$ from the variance $\langle \delta N \delta N' \rangle$.

Statistical Splitting and Gauge Symmetry

Scale Slider

Location: Ultrasoft Scale (g^2T)



The Split

Display the splitting of distribution and fields:

$$f(x, p, Q) = \bar{f} + \delta f$$

(Mean distribution + fluctuations)

$$A_\mu(x) = \bar{A}_\mu + a_\mu$$

(Mean field + field fluctuations)

$$\langle \delta f \rangle = \langle a_\mu \rangle = 0.$$

The Resulting Gauge Symmetry

Even after splitting, a mean background gauge symmetry remains:

$$\bar{A}'_\mu = U \left(\bar{A}_\mu + \frac{1}{g} \partial_\mu \right) U^{-1}$$

The fluctuations $\langle a_\mu \rangle = 0$ are fully compatible with this symmetry.

The Truncation of the Infinite Hierarchy

Scale Slider

Location: Ultrasoft Scale ($g^2 T$)

$$\langle \delta f \delta f \rangle$$



$$\langle \delta f \delta f \delta f \rangle$$

The Problem: An infinite hierarchy of differential equations relating n -th order correlators to $(n+1)$ -th order ones.

$$\langle \delta f \delta f \delta f \delta f \delta f \rangle \dots$$

The Solution: 2nd Moment Approximation

Assume small fluctuations ($g \ll 1$) and weakly correlated systems.

Result: The hierarchy is violently cut down to a closed, solvable set of differential equations for the mean quantities and their variances.

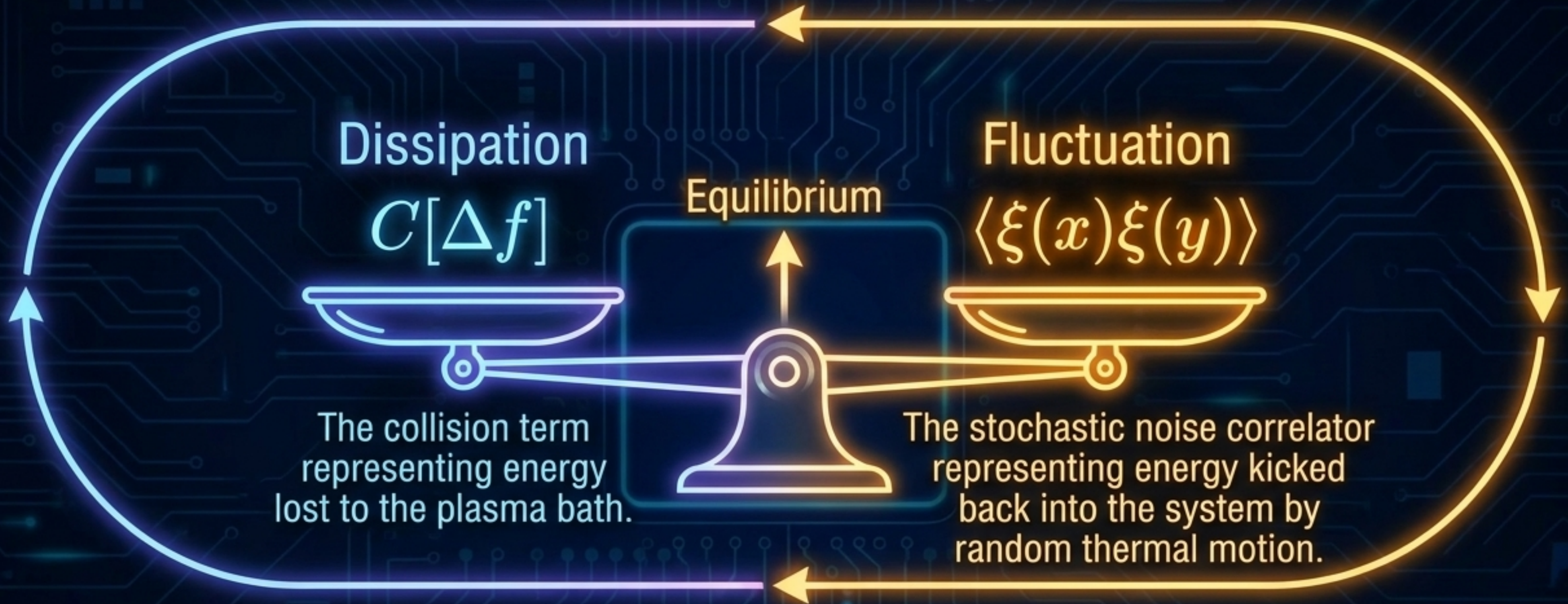
$$\langle \eta \rangle = \eta,$$

$$\langle \xi \rangle = \xi,$$

$$\langle J_{\text{fluc}}^\mu \rangle = J_{\text{fluc}}^\mu,$$

$$\left(D^2 a^\nu - \bar{D}^\nu (\bar{D} \cdot a) \right)_a + 2g f_{abc} \bar{F}_b^{\mu\nu} a_{c,\nu} = \delta J_{\text{fluc},a}^\mu(x)$$

The Fluctuation-Dissipation Theorem



Central Concept: At equilibrium, entropy S reaches a maximum. If you know the dissipation (the kinetic coefficient γ), the noise correlator is strictly fixed to balance it and maintain equilibrium (Brownian motion principle).

Warning: Classical Field Limit $\neq$ Classical Statistics Limit!

Classical (Maxwell)

$$f \sim e^{-(p_0 - \mu)/T}$$

Valid only for rarefied systems where fugacity $z \ll 1$ and interparticle distance $\bar{r} \gg \lambda_{\text{de Broglie}}$.

Quantum (Bose-Einstein)

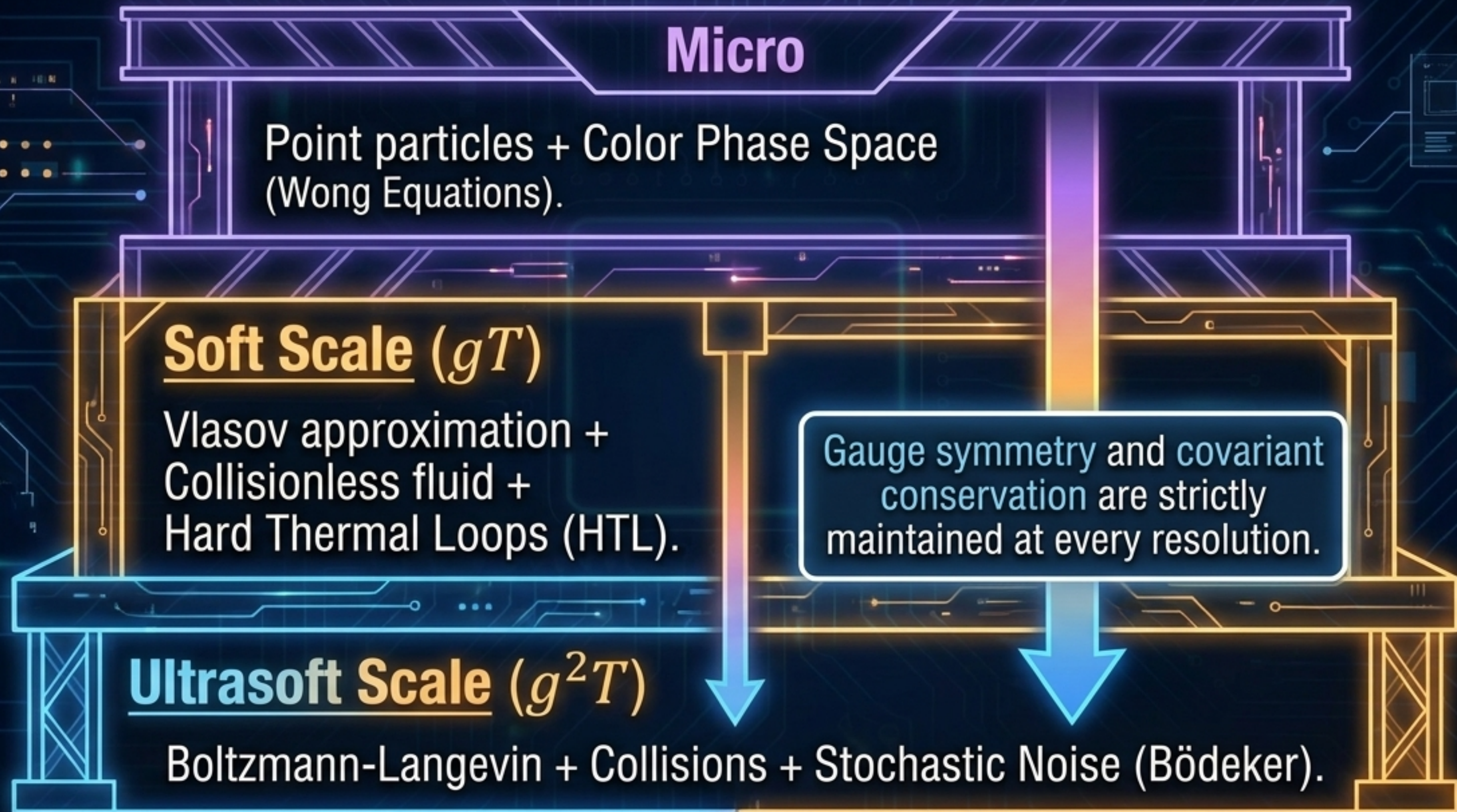
$$f_{\text{BE}} = \frac{1}{e^{(p_0 - \mu)/T} - 1}$$

Required when quantum statistics cannot be ignored.



The Gluon Problem: For gluons, chemical potential $\mu = 0$. Therefore, $z = 1$.
Quantum statistics cannot be ignored for gluons! Correlators must use the full f_{BE} term.

The Architectural Stack of Non-Abelian Transport Theory



Open Frontiers and the Construction Project



Beyond Leading Order

How do we construct transport equations at Next-to-Leading Order (NLO)?



Colorless Fluctuations

Computed only to leading order without a proper derivation of collision terms. Does this require numerical computation?



Color Superconductors

Full transport coefficients for the gapped phases remain uncalculated.



Out of Equilibrium

How does this entire mathematical apparatus function far from thermal equilibrium?