

The high temperature QCD static potential

Cristina Manuel



TUM, 17th October 2025

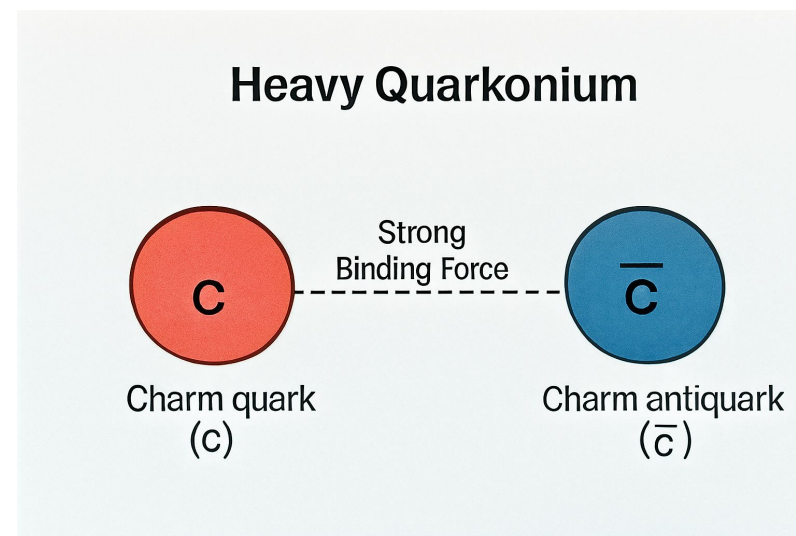
Margaret Carrington, C.M., Joan Soto, 2407.00310, 2509.24746

Outline

- Motivation of this work
- Thermal QCD
- Thermal corrections to the QCD Static Potential
- Comparison with lattice
- Conclusions

Introduction

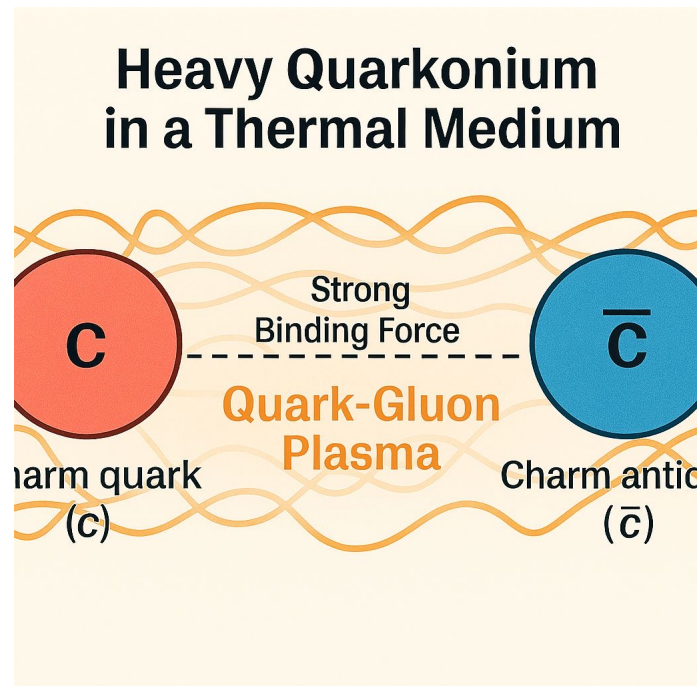
The static potential gives the **dominant** contribution between heavy quark and antiquark at low E



- At $T=0$ It is attractive and Coulomb like at small r (and linearly rising at large r)

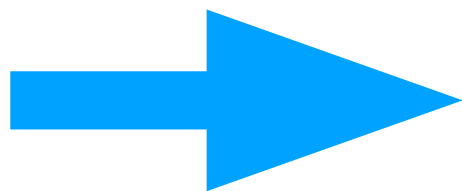
$$V^{(0)}(r) = -\frac{C_F g^2}{4\pi r} + \sigma r$$

- At high T, it is a Yukawa-like potential, with screening mass $\sim gT$



$$V_T^{(0)}(r) = -g^2 \frac{C_F e^{-m_D r}}{4\pi r}$$

J/Psi suppression at high T Matsui and Satz, '86, as bound states melt



The potential develops an imaginary part
Laine, Philipsen, Romatschke, Tassler, '07

(**damping** instead of **melting** of bound states)

Goal

Push the initial high T computations of the static potential

(computations were carried out only to leading order)

Provide physical motivated forms of the potential V , information that might be of great help for lattice

Thermal QCD and perturbative computations

Not easy ... main difficulties :

- Thermal Field Theory : use either Imaginary Time Formalism (sum over Matsubara frequencies), or Real Time Formalism (doubling of dof)

(thermodynamical or static properties seem easier using ITF)

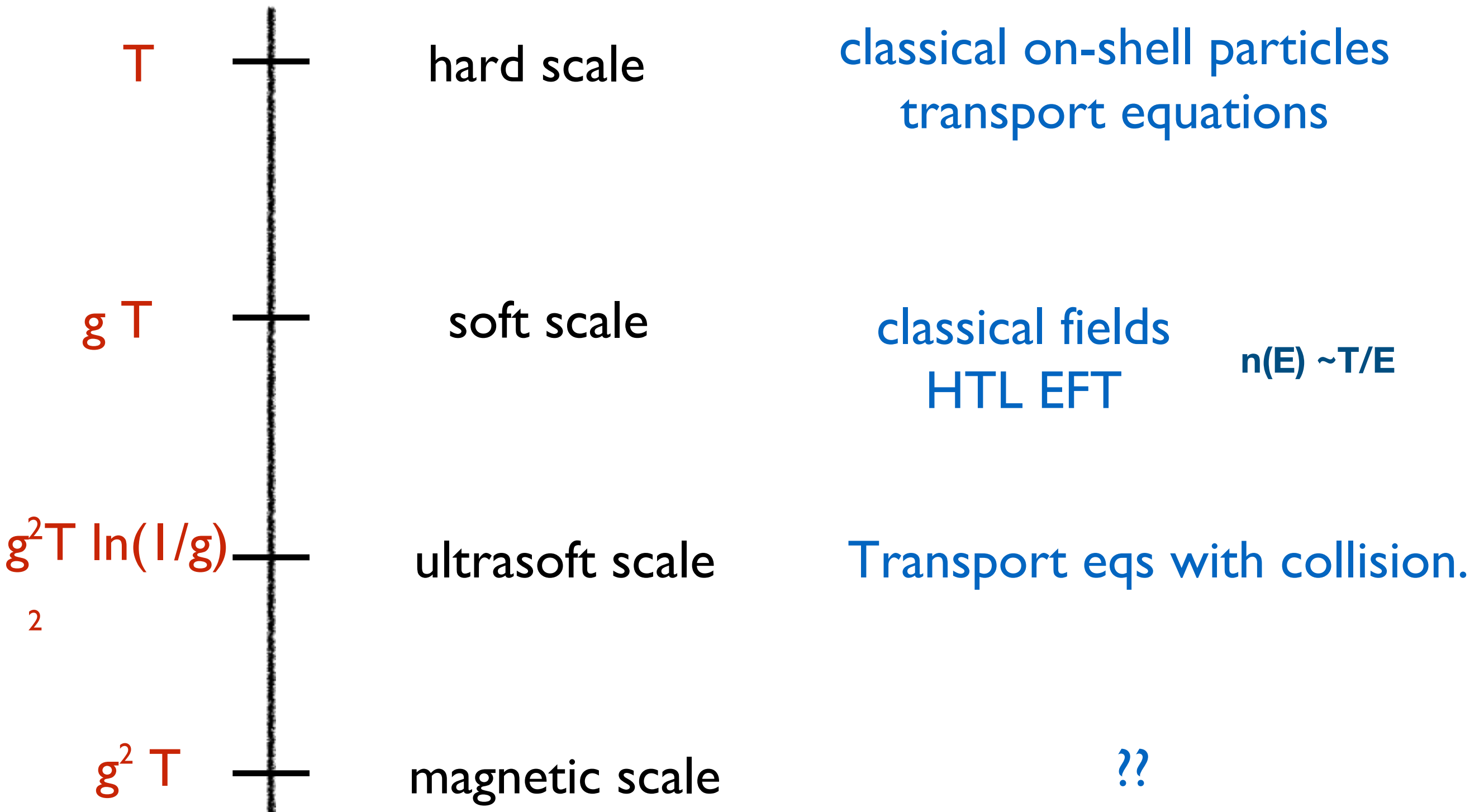
- Expanding in loops $\neq$ expanding in couplings

(good arena for the use of EFT techniques ... not easy for dynamical properties)

- There is a full non-perturbative scale, the magnetic scale

Hot QCD plasmas

$$g \ll 1$$

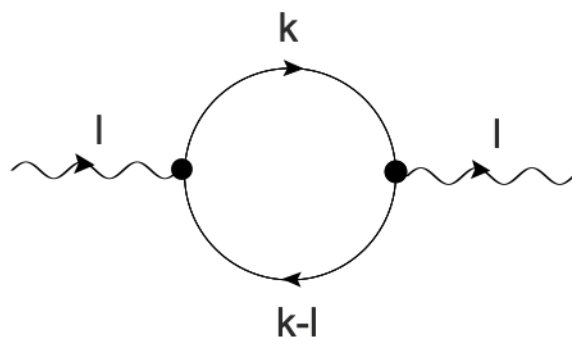


Breakdown of perturbation theory

Braaten and Pisarski; Frenkel and Taylor, 90'

At high temperature: two relevant scales $\left\{ \begin{array}{l} \text{hard} \sim T \\ \text{soft} \sim gT \end{array} \right.$
 $g \ll 1$

One-loop thermal corrections **hard thermal loops (HTLs)**
as relevant as the tree amplitudes for **soft momenta**
(and they arise from **hard** loop momenta)



$$\Pi_{\text{HTL}}(l) \sim g^2 T^2$$

$$\frac{\Pi_{\text{HTL}}(l)}{l^2} \sim 1$$

for soft momentum

and have to be **resummed** into effective vertices and propagators

How to improve the previous picture?

let us start with QED

- Perturbative corrections (two-loop diagrams)

$$e^2 \times (\text{HTL})$$

- Power corrections (POW)

$$\left(\frac{k}{T}\right)^2 \times \text{HTL}$$

$$k \ll T \Rightarrow e^2 \times \text{HTL}$$

From one-loop diagrams: expanding (soft/hard)

[arXiv:1712.07949](#)

From transport theory: pushing the gradient expansion, keeping quantum corrections

[arXiv:2107.03655](#)

$$\Delta^2 = (\partial_x \cdot \partial_q)^2 \quad (\hbar^2 \text{ corrections})$$

Using effective field theory methods (OSEFT): expansion done at the Lagrangian level

[arXiv:1603.05514](#)

(I am ignoring possible effects of P and CP violation in this talk!)

Power corrections to the HTL photon polarization tensor

We use the RTF in the Keldysh representation

$$S_{R/A}(Q) = \frac{\not{Q}}{Q^2 \pm i \text{sgn}(q_0)\eta} \quad S_S(Q) = -2\pi i \not{Q} (1 - 2n_F(|q_0|))\delta(Q^2)$$

$$\eta \rightarrow 0^+ \quad Q^\mu = (q_0, \mathbf{q})$$

$$\Pi_R^{\mu\nu}(L) = -\frac{ie^2}{2} \int \frac{d^4 q}{(2\pi)^4} \left(\text{Tr}[\gamma^\mu S_S(K) \gamma^\nu S_R(Q)] + \text{Tr}[\gamma^\mu S_A(K) \gamma^\nu S_S(Q)] \right)$$

$$K = Q - L$$

$$\Pi_R^{\mu\nu}(L) = e^2 \int \frac{d^3 q}{(2\pi)^3} \frac{1 - 2n_F(q)}{q} \left(\frac{2qv^\mu v^\nu - (v^\mu L^\nu + v^\nu L^\mu) + g^{\mu\nu} v \cdot L}{v \cdot L - \frac{L^2}{2q} + i \text{sgn}(q - l_0)\eta} \right. \\ \left. - \frac{2q\tilde{v}^\mu \tilde{v}^\nu - (\tilde{v}^\mu L^\nu + \tilde{v}^\nu L^\mu) + g^{\mu\nu} \tilde{v} \cdot L}{\tilde{v} \cdot L + \frac{L^2}{2q} + i \text{sgn}(q + l_0)\eta} \right)$$

$$v^\mu = (1, \mathbf{q}/q) \quad \tilde{v}^\mu = (1, -\mathbf{q}/q)$$

Expanding the integrand: use dimensional regularization!

$$d = 3 + 2\epsilon$$

$$\Pi_{(1)}^{\mu\nu}(L) = 2e^2 \nu^{3-d} \int \frac{d^d q}{(2\pi)^d} \frac{1 - 2n_F(q)}{q} \left(\frac{v^\mu v^\nu L^2}{(v \cdot L)^2} - \frac{v^\mu L^\nu + v^\nu L^\mu}{v \cdot L} + g^{\mu\nu} \right) ,$$

This is the HTL

$$\Pi_{(3)}^{\mu\nu}(L) = 2e^2 \nu^{3-d} \int \frac{d^d q}{(2\pi)^d} \frac{1 - 2n_F(q)}{q^3} \frac{L^4}{4(v \cdot L)^2} \left(\frac{v^\mu v^\nu L^2}{(v \cdot L)^2} - \frac{v^\mu L^\nu + v^\nu L^\mu}{v \cdot L} + g^{\mu\nu} \right) ,$$

First power correction

Higher order power corrections

$$\frac{(1 - 2n_f(q))}{q} \left(\frac{L^4}{4q^2(v \cdot L)^2} \right)^n \times \text{int}_{\text{HTL}}$$

Effective Lagrangian

Carignano, CM, Soto, '18

$$\mathcal{L}_{\text{HTL}}^{(1)} = \frac{e^2}{2} \int \frac{d^3 q}{(2\pi)^3} \left\{ \frac{2n_F(q)}{q} \left(F_{\rho\alpha} \frac{v^\alpha v^\beta}{(v \cdot \partial)^2} F_\beta^\rho \right) - \frac{2(n_F(q) + n_B(q))}{q} \left(\bar{\psi} \frac{v \cdot \gamma}{(iv \cdot D)} \psi \right) \right\}$$

In d spatial dimensions

$$v^\mu = q^\mu / |\mathbf{q}|$$

$$\mathcal{L}_{\text{HTL}}^{(3)\gamma} = \frac{e^2 \nu^{3-d}}{4} \int \frac{d^d q}{(2\pi)^d} \frac{1 - 2n_F(q)}{q^3} \left\{ F_{\rho\alpha} \frac{v^\alpha v^\beta}{(v \cdot \partial)^4} \partial^4 F_\beta^\rho \right\}$$

$$\begin{aligned} \mathcal{L}_{\text{HTL}}^{(3)\psi} = & \frac{e^2 \nu^{3-d}}{4} (d-1) \left[\int \frac{d^d q}{(2\pi)^d} \frac{n_F(q) + n_B(q)}{q^3} \left\{ \bar{\psi} D^2 \frac{v \cdot \gamma}{(iv \cdot D)^3} D^2 \psi \right\} \right. \\ & \left. + \int \frac{d^d q}{(2\pi)^d} \frac{1 + 2n_B(q)}{2q^3} \left\{ \bar{\psi} \left(D^2 (iD \cdot \gamma) \frac{1}{(iv \cdot D)^2} + \frac{1}{(iv \cdot D)^2} (iD \cdot \gamma) D^2 \right) \psi \right\} \right] + \mathcal{O}(e^3) \end{aligned}$$

New pieces: corrections to the soft propagation.

For soft momentum (schematically) $L \sim eT$

$$\Pi^{(1-loop)} \sim e^2 T^2 \left(1 + \frac{L^2}{T^2} + \frac{L^4}{T^4} + \dots \right)$$

HTL POW

The first POW competes with 2-loops coming from hard scales for soft momenta

$$\Pi^{(2-loops)} \sim \alpha^2 T^2$$

Nothing new: loop expansion $\neq$ perturbative expansion

But POW are the **leading** correction to the HTL for $eT \ll L \ll T$

Two-loops corrections have been also computed

Carignano, Carrington, Soto 2019; Gorda et al 2022

$$\Pi_I = \Pi_I^{\text{htl}} + \Pi_I^{\text{pow}\cdot\text{corr}} + \Pi_I^{2\text{loop}}, \quad I = L, T$$

$$\Pi_L^{\text{htl}}(l_0, \mathbf{l}) = \frac{e^2 T^2}{3} \left(1 - \frac{l_0}{2|\mathbf{l}|} \ln \left(\frac{l_0 + |\mathbf{l}|}{l_0 - |\mathbf{l}|} \right) \right)$$

In Feynman gauge

$$\Pi_L^{\text{pow}}(l_0, \mathbf{l}) = -\frac{e^2}{4\pi^2} \left(\mathbf{l}^2 - \frac{l_0^2}{3} \right) \left(1 - \frac{l_0}{2|\mathbf{l}|} \ln \left(\frac{l_0 + |\mathbf{l}|}{l_0 - |\mathbf{l}|} \right) \right)$$

$$\Pi_L^{2\text{loop}}(l_0, \mathbf{l}) = \frac{e^4 T^2 L^2}{8\pi^2 \mathbf{l}^2}$$

$$\Pi_T^{\text{htl}}(l_0, \mathbf{l}) = \frac{e^2 T^2}{3} \frac{l_0}{4\mathbf{l}^3} \left(2|\mathbf{l}|l_0 - L^2 \ln \left(\frac{l_0 + |\mathbf{l}|}{l_0 - |\mathbf{l}|} \right) \right)$$

$$\Pi_T^{\text{pow}}(l_0, \mathbf{l}) = \frac{e^2}{4\pi^2} \left(\frac{l_0^2}{2} + \frac{l_0^4}{6\mathbf{l}^2} - \frac{2\mathbf{l}^2}{3} - \frac{l_0^3}{12\mathbf{l}^3} \left(2\mathbf{l}^2 + l_0^2 - \frac{3\mathbf{l}^4}{l_0^2} \right) \ln \left(\frac{l_0 + |\mathbf{l}|}{l_0 - |\mathbf{l}|} \right) \right)$$

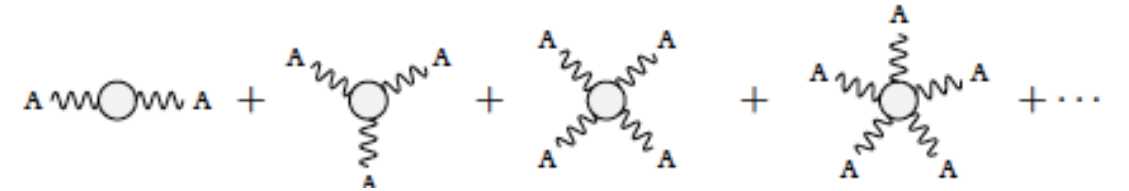
$$\Pi_T^{2\text{loop}}(l_0, \mathbf{l}) = -\frac{e^4 T^2}{16\pi^2} \frac{l_0}{|\mathbf{l}|} \ln \left(\frac{l_0 + |\mathbf{l}|}{l_0 - |\mathbf{l}|} \right)$$

Taking the renormalization scale as $\nu = T e^{-\gamma_E/2-1} \sqrt{\pi}/2$

HTL theory in QCD and corrections

Not only the propagators are modified, also effective (non-local) propagators are needed

$$S_{\text{HTL}}^+ = -\frac{m_D^2}{4} \int_{x,v} \text{Tr} \left(F_{\alpha\mu} \frac{v^\alpha v^\beta}{(v \cdot D)^2} F_\beta{}^\mu \right)$$



Corrections to HTL physics

- Power corrections and two-loop contributions have been computed in covariant gauges ; they are gauge dependent though

Gorda, Paatelainen, Säppi, Seppänen 2307.08734

Ekstedt, 2302.04894

- But one-loop diagrams with soft momentum **are more important** but hard to compute

$$n_B(E) \sim \frac{T}{E} \sim \frac{1}{g}$$

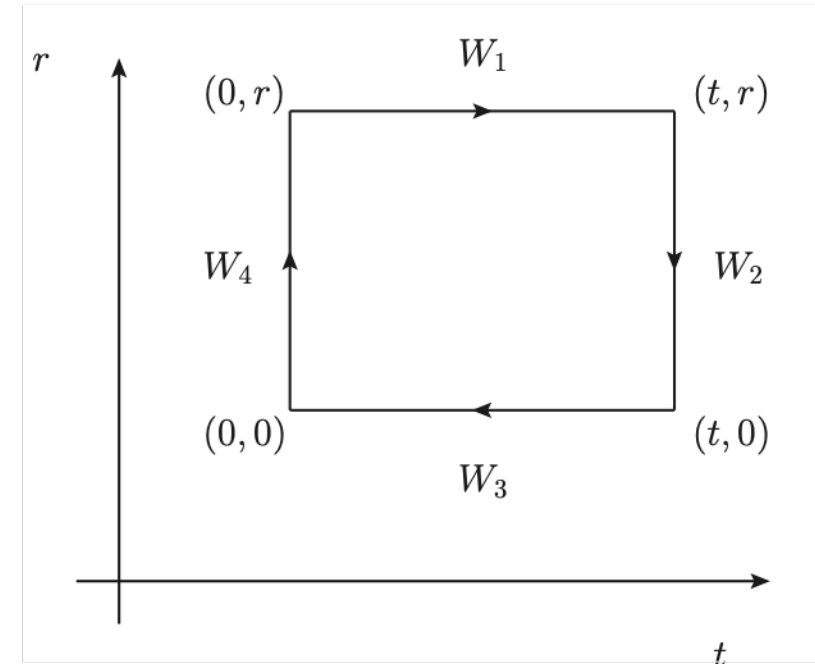
Need resummed HTL theory

The static potential

Potential obtained from the QCD Wilson loop

$$V(\mathbf{r}) = \lim_{t \rightarrow \infty} \frac{i}{t} \ln[W(t, \mathbf{r})]$$

$$W = \mathcal{P} \text{Exp} \left(ig \oint dx^\mu A_\mu(x) \right)$$



(similar results if we apply standard diagrams in p space)

- Heavy quarks with $M \gg$ any other scale, unthermalized
- Thermal plasma, at high T - we use the real time formalism of TFT
- Computation carried out in Coulomb gauge; use of dimensional regularization

Leading order potential

Obtained from the time ordered longitudinal gluon propagator, when $p \ll T$

In momentum space

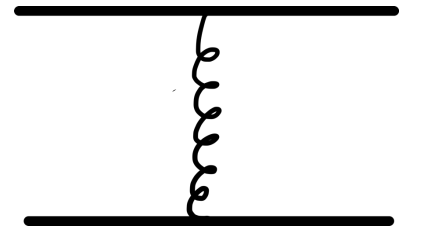
$$C_F = (N_c^2 - 1)/(2N_c)$$

$$V_{1\text{lo}}(p) = g^2 C_F G(0, p)$$

$$G(0, p) = -\frac{1}{m_D^2 + p^2} + \frac{i\pi T m_D^2}{p (m_D^2 + p^2)^2}$$

$$m_D^2 = \frac{g^2 T^2 (N_c + N_f/2)}{3}$$

Debye mass squared



$$V_{1\text{lo}}(r) = -\frac{g^2 C_F}{4\pi \hat{r}} \left(m_D e^{-\hat{r}} - 2iT I_2(\hat{r}) \right)$$

$$\hat{r} = r m_D$$

Note: when screening is important $p \sim m_D$

$$\text{Im}(V_{110}) \gg \text{Re}(V_{110})$$

And then narrow resonances cannot exist

$$\text{Im}(V_{110}) \sim \text{Re}(V_{110}) \quad \text{for} \quad p_d \sim (m_D^2 T)^{1/3} \sim g^{2/3} T$$

Narrow resonances exist if the typical momentum is semi-hard

$$m_D \ll p \ll T$$

Semi-hard momenta

$$p \sim g^a T$$

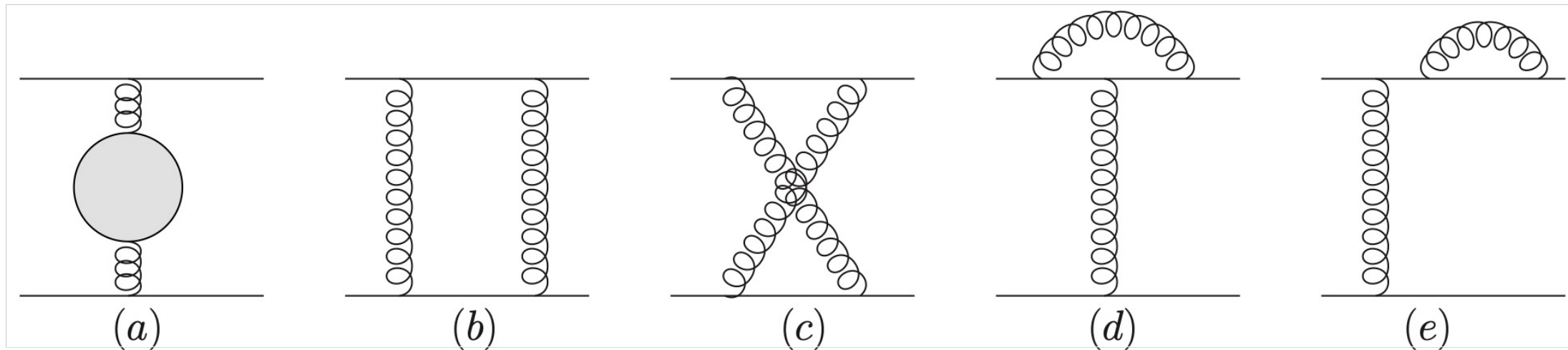
$$1/2 < a < 2/3$$

$$\text{Re}[\tilde{V}_{1LO}(p)] \sim \frac{g^{2-2a}}{T^2}$$

$$\text{Im}[\tilde{V}_{1LO}(p)] \sim \frac{g^{4-5a}}{T^2}$$

Potential beyond LO

Diagrams needed in the computation

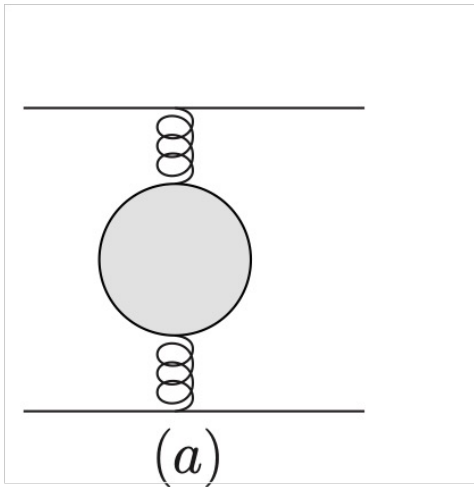


We use the Hard Thermal Loop (HTL) effective field theory in the computation

(Braaten and Pisarski, '92)

When the momentum is soft $\sim gT$, resummed (and non-local) propagators and vertices have to be used

But (fortunately!) for semi-hard momentum enormous simplifications arise



Gluon polarization tensor for semi-hard external momenta MUCH easier
that for soft external momenta

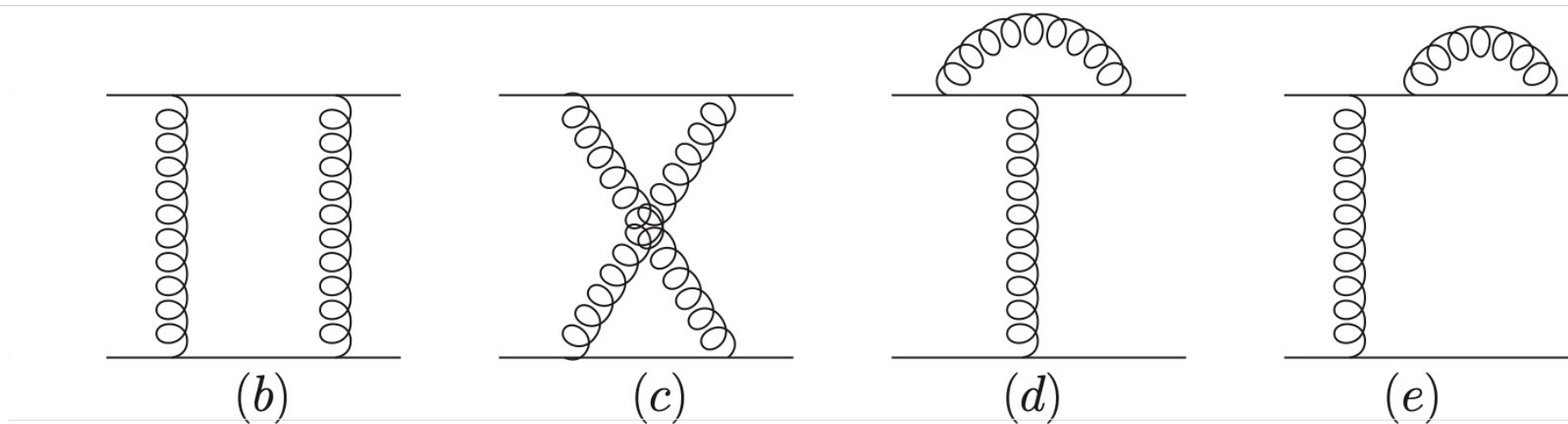
(in Coulomb gauge)

Correction to the HTL from soft region

$$[\Pi_{00}(p_0 \rightarrow 0, p)]_{\text{semi-hard}} = -\frac{g^2 N_c T}{4} \left(p + \frac{7i}{3\pi} p_0 \right) + \mathcal{O}(p_0^2) .$$

Power correction

$$\text{Im } \Pi^{\text{POW}}(p_0, p) \approx \frac{g^2}{8\pi} \left(N_c - \frac{N_F}{2} \right) p_0 p$$



In all the above diagrams p is semi-hard, k the internal momentum

These diagrams are dominated by $k \sim m_D$ (use of the HTL effective theory !)

But these diagrams can be computed in an expansion in k/p and m_D/p

Fully expanded potential in momentum space

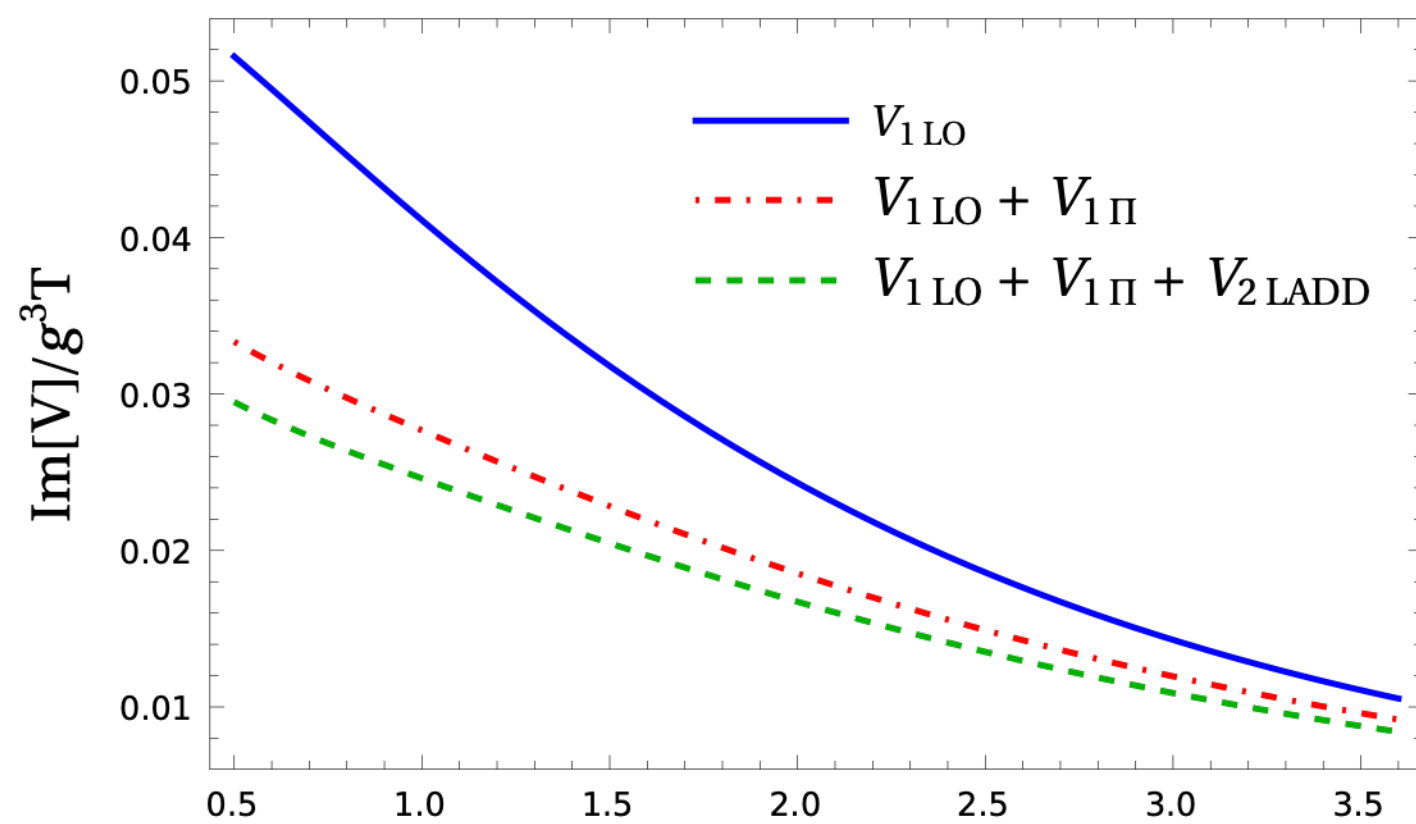
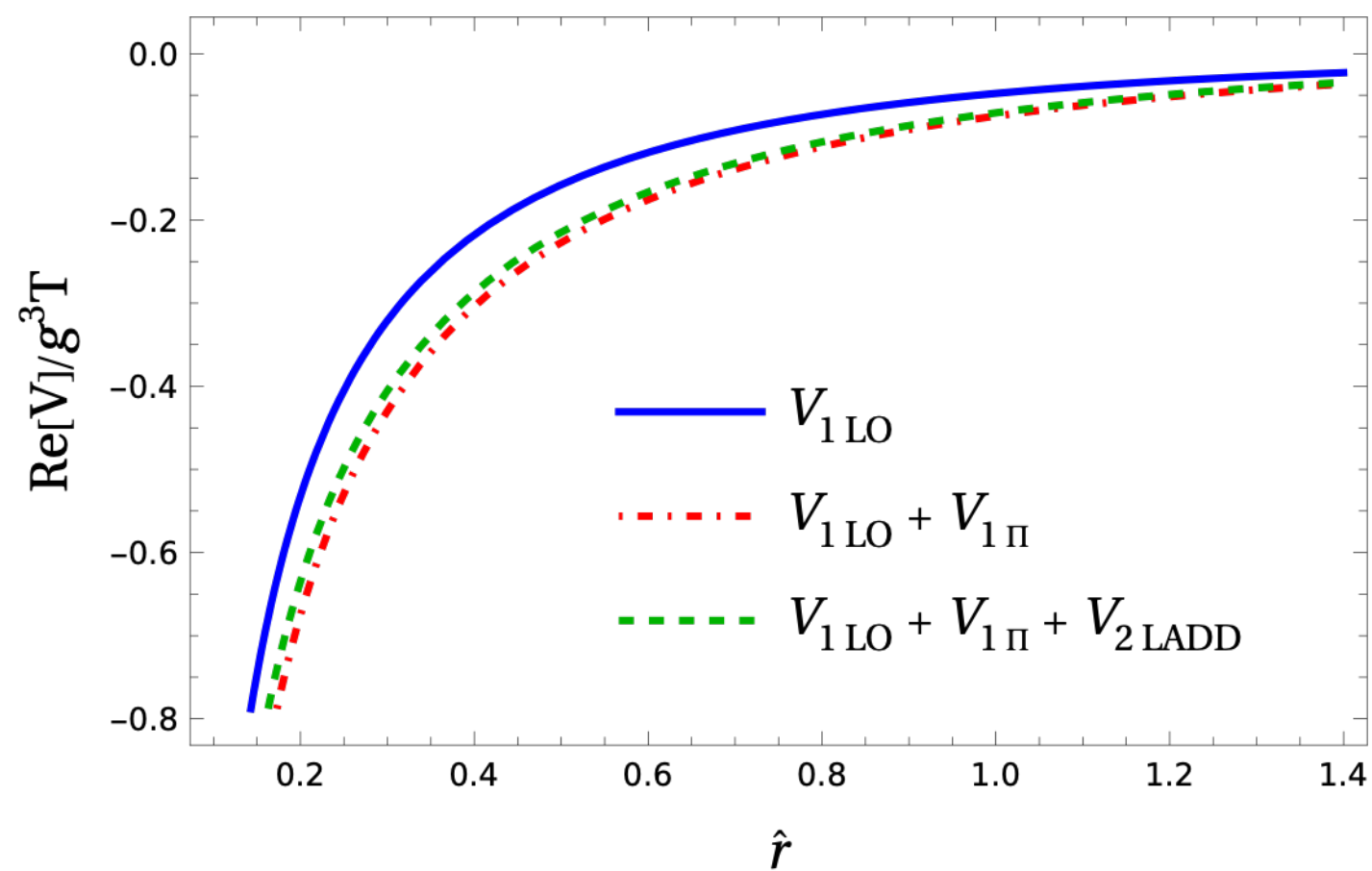
$$V_{2,\text{exp}}(p) = -\frac{g^4 C_F N_c T}{16\pi m_D p^2} \left(1 - \frac{3\pi^2}{16} + \frac{4\pi m_D}{p} + \frac{m_D^2}{p^2} \left(\frac{5\pi^2}{24} - \frac{4}{3} \right) \right) \\ - i \frac{g^4 C_F T^2}{16p^4} \left(N_c \left(\frac{56}{3\pi} - \left(1 - \frac{3\pi^2}{16} \right) \frac{m_D}{p} \right) - \frac{4}{\pi} \left(N_c - \frac{N_f}{2} \right) \frac{p}{T} \right)$$

For $p = g^a T$ $1/2 < a < 2/3$

Omitted corrections to the real part of order g^{4-2a}

to the imaginary part of order g^{6-6a}

$$g = 1.8$$



Keeping the momentum semi-hard: in coordinate space the potential is expected to be valid only for

$$rm_D \ll 1 \ll rT$$

Damped approximation:
keep in the gluon propagators unexpanded factors $1/(p^2 + m_D^2)$

which is more realistic for momenta getting close to the soft scale

In the damped approximation

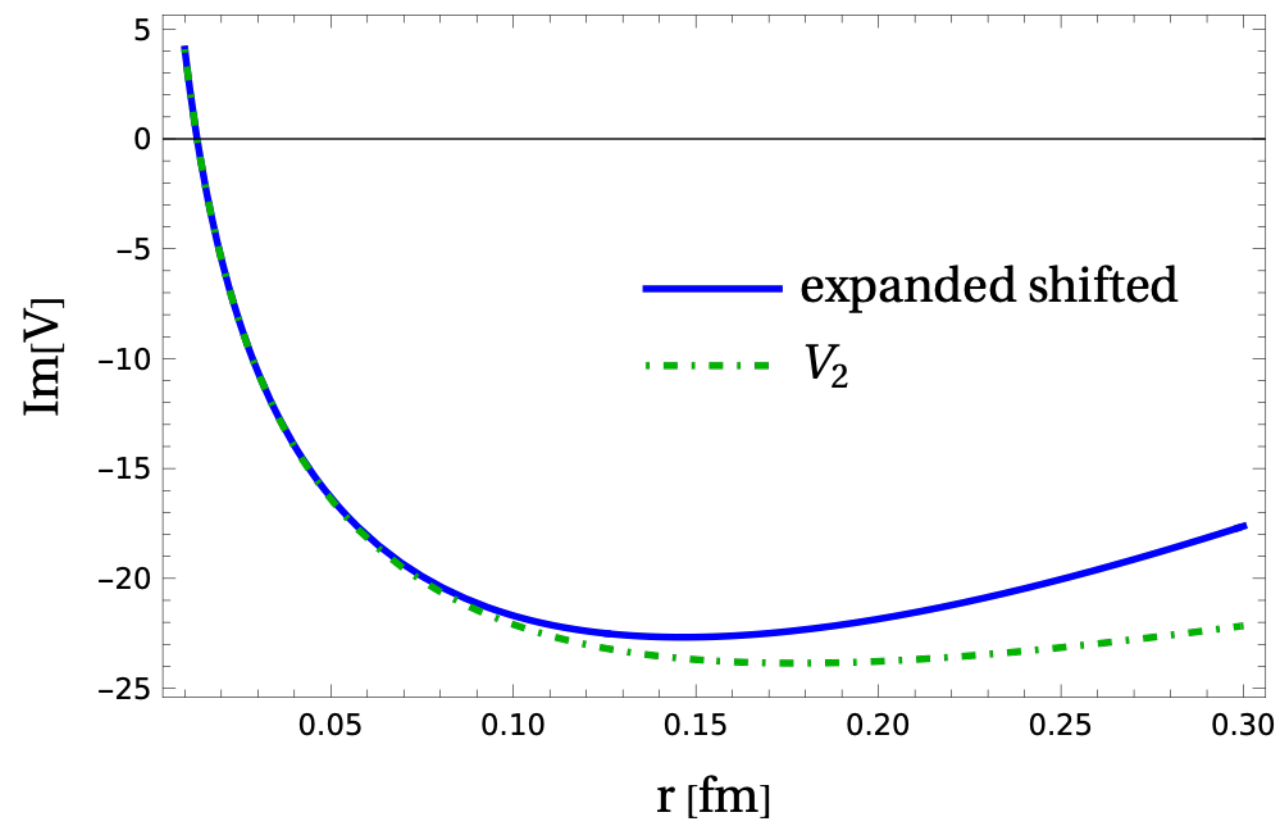
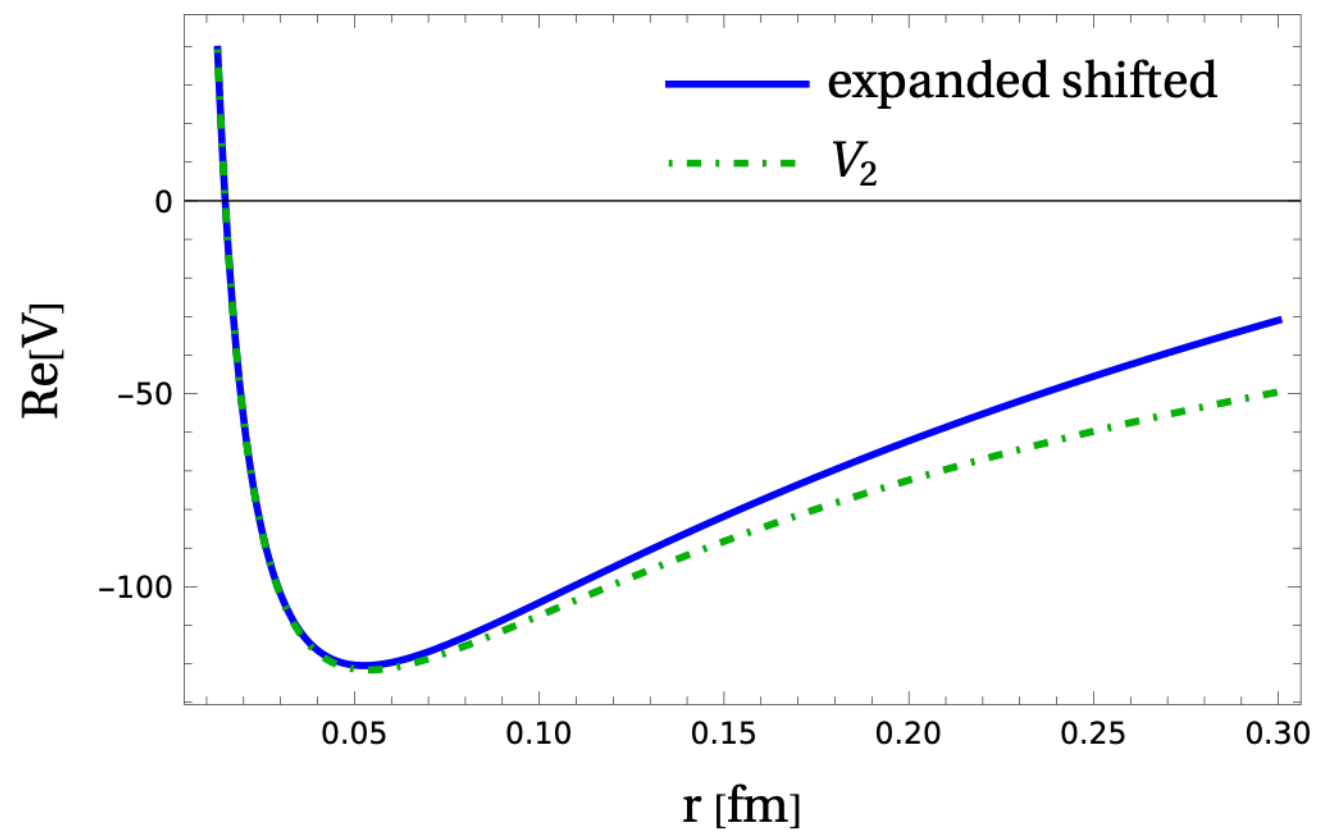
$$V_{1\text{lo}}(r) = -\frac{g^2 C_F}{4\pi\hat{r}} \left(m_D e^{-\hat{r}} - 2iT I_2(\hat{r}) \right) \quad \hat{r} = rm_D$$

$$\text{Re}[V_2] = \frac{g^4 N_c C_F T}{64\pi^2 \hat{r}} \left\{ 8 (I_2(\hat{r}) - I_1(\hat{r})) + \frac{e^{-\hat{r}}}{16} \left(3\pi^2 - 16 + \frac{\hat{r}}{6} (16 - \pi^2) \right) \right\}$$

$$i\text{Im}[V_2] = -i \frac{g^3 C_F T}{16\pi^2 \hat{m}_D} \left\{ \frac{3\pi^2 - 16}{32\hat{r}} I_2(\hat{r}) + \frac{7}{3} N_c e^{-\hat{r}} \right. \\ \left. - \frac{2g\hat{m}_D}{\pi\hat{r}} \left(N_c - \frac{N_f}{2} \right) (I_1(\hat{r}) - I_2(\hat{r})) \right\}$$

$$m_D = gT\hat{m}_D$$

$$I_j(\hat{r}) = \int_0^\infty d\hat{p} \sin(\hat{p}\hat{r}) (\hat{p}^2 + 1)^{-j}$$



Comparison with lattice data

Our potential in p space describes the semi-hard scales

The damped approximation allows us to get closer to $r \sim 1/m_D$

For scales $r \ll 1/m_D$ we miss the contribution of the soft modes $p \sim m_D$

but these have a universal form $\sim$ a polynomial in r (up to logs), as

one can expand in the Fourier transform $e^{i\vec{p}\cdot\vec{r}}$

We add contributions to the coordinate potential arising from the soft modes

$$\text{Re}[V_{\text{soft}}] = C + g^3 q_0 T$$

$$\text{Im}[V_{\text{soft}}] = g^3 i_0 T + g^5 i_2 r^2 T^3$$

C adjusts the origin of energies;

We obtain the coefficients by fitting to lattice

$$C = 209.5 \text{ MeV}$$

Potential [1]. [Bzavov, Hoying, Larsen, Mukherjee, Petreczky, Rothkopf, Weber, 23](#)

Mass spectrum and widths [2] [Larsen, Meine, Mukherjee, Petreczky, 20](#)

Fourier transform

$$V_{2,exp}(r) = -\frac{g^4 C_F N_c T}{16\pi m_D} \left\{ \left[\left(1 - \frac{3\pi^2}{16}\right) \frac{1}{4\pi r} - \frac{m_D}{\pi} \ln(r^2 m_D^2) - \left(\frac{5\pi^2}{24} - \frac{4}{3}\right) \frac{r m_D^2}{8\pi} \right] \right. \\ \left. - i\pi T m_D \left[\frac{7r}{3\pi^2} - \left(1 - \frac{3\pi^2}{16}\right) \frac{m_D r^2}{24\pi^2} (1 - \ln(r^2 m_D^2)) - \frac{1}{\pi^3 T} \left(1 - \frac{N_f}{2N_c}\right) \ln(r^2 m_D^2) \right] \right\}$$

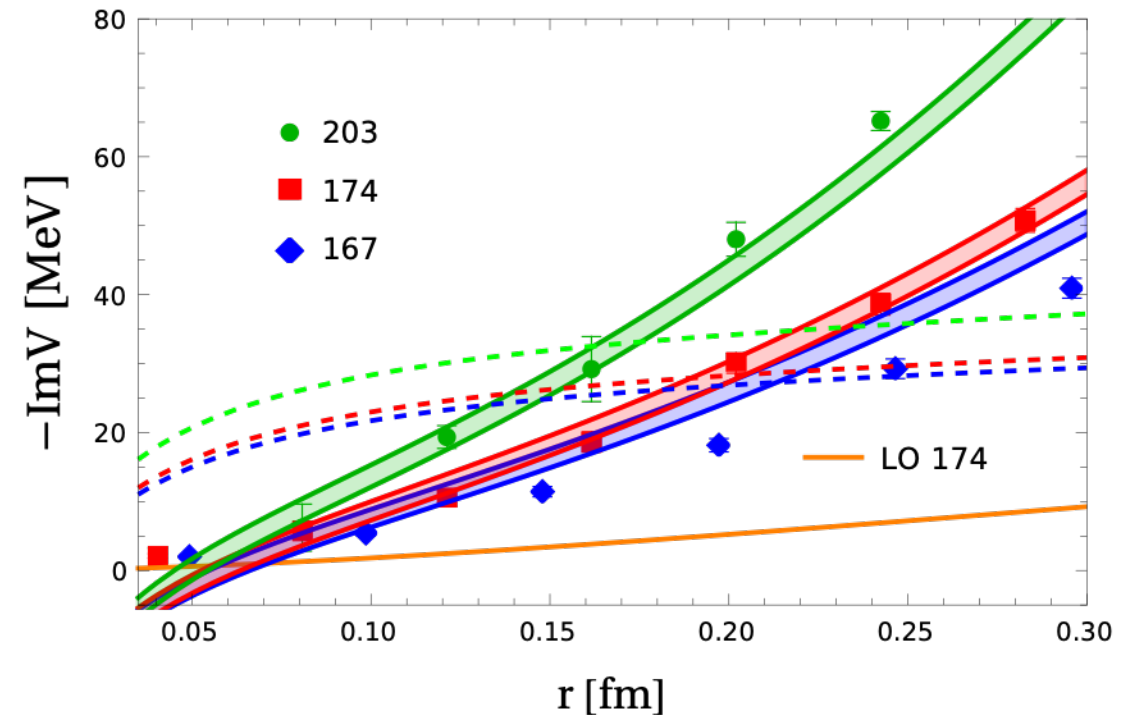
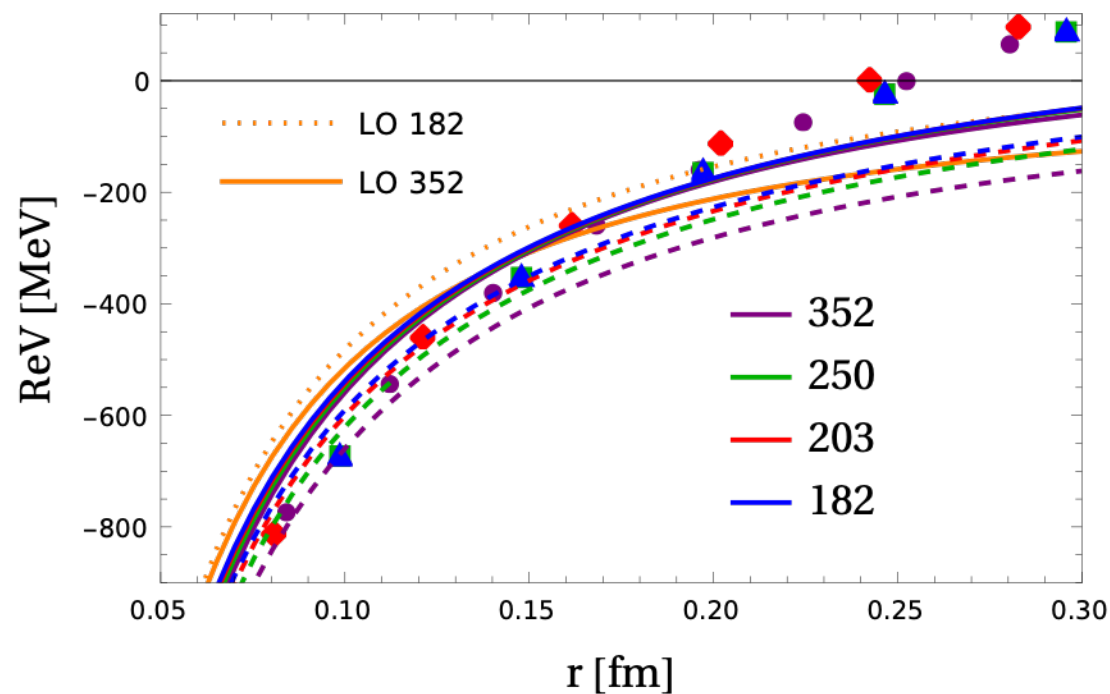
IR problems arise : divergencies can be absorbed in the soft terms

$$V_{2\text{soft}}(r) = g^4 \bar{q}_0 T + i \left(g^3 \bar{i}_0 T + g^5 \bar{i}_2 r^2 T^3 \right)$$

Use $g = 1.8$ from fit $T=0$ lattice data; bottom quark mass $M = 4676$ MeV

Find fitted constants to all available T (solid bands are uncertainties in the fitting with lattice [1])

$$(q_0, i_0, i_2) = (0.027, -0.019 \pm 0.001, 0.194 \pm 0.002)$$



The real part of V depends very little on T (like the lattice data)

The imaginary part of the V has big contribution from the soft region

We solve the Schrödinger equation using the real part of the potential and find the binding en

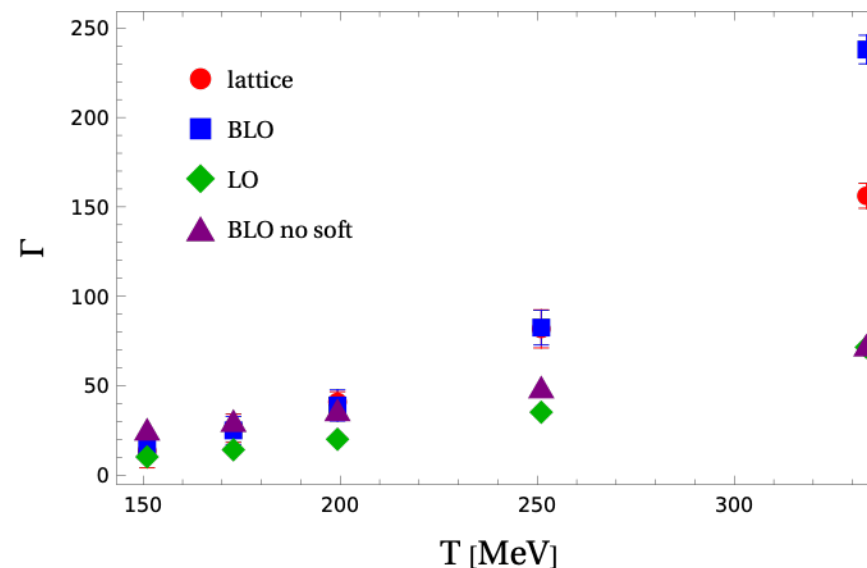
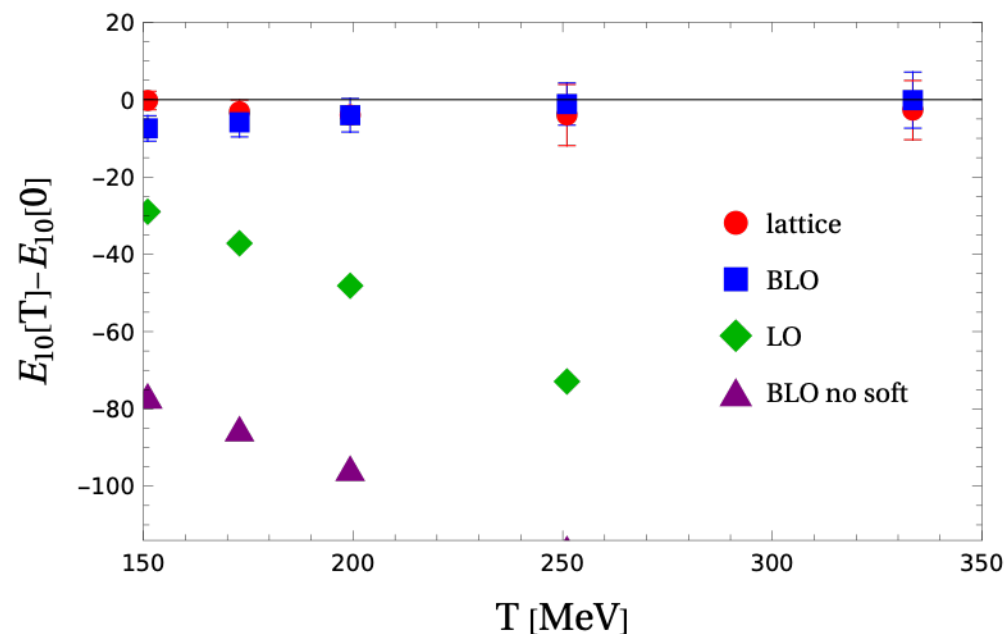
Decay rates are found as

$$\Gamma = -2\langle \text{Im}V \rangle$$

Fitting coefficients found by fitting with data in [2] (C plays no role ...)

$$(q_0, i_0, i_2) = (0.044 \pm 0.002, -0.0026 \pm 0.009, 0.052 \pm 0.002)$$

Big discrepancies with [1] of i_2



Dissociation Temperature

It can be defined as the temperature at which the binding energies equal the decay width

At LO

$$T_d = 193 \text{ MeV}$$

BLO, no soft

$$T_d = 210 \text{ MeV}$$

BLO, with the fit to [1]

$$T_d = 142.7 \pm 1.1 \text{ MeV}$$

BLO, with the fit to [2]

$$T_d = 202 \pm 10 \text{ MeV}$$

(due to the different values in fitting coefficient; it suggests a problem with the data)

CONCLUSIONS

- We have computed corrections to the LO static potential in QCD in momentum space , valid in the regime $m_D \ll p \ll T$
- We have provided the static potential in configuration space, including pieces due to soft modes (which are universal) that we fit with lattice data
- Better description than LO of lattice data